

Exercise 46

For the following exercises, use a calculator to help answer the questions.

Evaluate $(1 + i)^k - (1 - i)^k$ for $k = 4, 8,$ and 12 . Predict the value if $k = 16$.

Solution

Evaluate the given expression for the desired values of k .

$$\begin{aligned}(1 + i)^4 - (1 - i)^4 &= (1 + 4i + 6i^2 + 4i^3 + i^4) - (1 - 4i + 6i^2 - 4i^3 + i^4) \\ &= (1 + 4i - 6 - 4i + 1) - (1 - 4i - 6 + 4i + 1) \\ &= (-4) - (-4) \\ &= 0\end{aligned}$$

$$\begin{aligned}(1 + i)^8 - (1 - i)^8 &= [(1 + i)^4]^2 - [(1 - i)^4]^2 \\ &= (-4)^2 - (-4)^2 \\ &= 0\end{aligned}$$

$$\begin{aligned}(1 + i)^{12} - (1 - i)^{12} &= [(1 + i)^4]^3 - [(1 - i)^4]^3 \\ &= (-4)^3 - (-4)^3 \\ &= 0\end{aligned}$$

$$\begin{aligned}(1 + i)^{16} - (1 - i)^{16} &= [(1 + i)^4]^4 - [(1 - i)^4]^4 \\ &= (-4)^4 - (-4)^4 \\ &= 0\end{aligned}$$